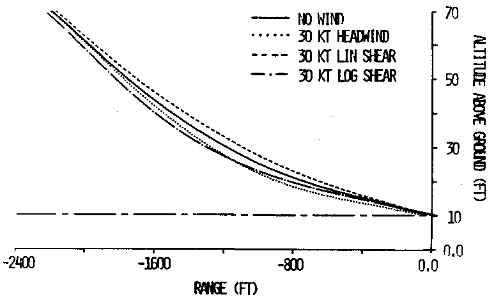

Fig. 1 Range trajectories with different initial conditions for various winds.

completely:

$$h = 70\exp(-0.255t)$$

It should be noted that d_2 was not included in the computation.

With the optimal control law specified, the no-wind state equation was integrated by implementing a fourth-order Runge-Kutta algorithm. The weights that resulted in the best adherence to the in-flight and terminal constraints are

$$S = \text{diag}[10^8 \ \ 0 \ \ 0 \ \ 10^6 \ \ 10^2 \ \ 10^2 \ \ 10^6 \ \ 10^6 \ \ 0]$$

$$Q = \text{diag}[10^4 \ \ 0 \ \ 10^4 \ \ 10^6 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 10^4]$$

$$R = \text{diag}[5\times 10^6 \ \ 10^3]$$

Based on the no-wind optimal solution, $F(t)$ and $v(t)$ were tabulated with respect to the range associated with each time interval, $F(r)$ and $v(r)$. Simulation of flare in wind conditions then was accomplished with the following equations:

$$\dot{x} = Ax + Bu + d_1 + d_2$$

$$u = F(r)x + v(r)$$

where r is the state specified in the state vector x. The disturbance vector d_1 includes ground effect, which is a function of altitude. The wind disturbance vector d_2 also is specified as a function of altitude.

It should be noted that the in-flight and terminal constraints, along with their weights, were not fully determined prior to simulation with winds. Simulation in wind conditions aided in identifying the constraints and weights that were selected.

Results

The optimal flare was simulated on a CDC 6600 digital computer. Initial conditions for the flare were obtained from the "Speckled Trout" C-135 hybrid simulation mentioned in the Introduction; the state vectors at flare initiation for the no-wind, 30-knot constant headwind, 30-knot linear headwind shear, and 30-knot logarithmic headwind shear were used. Stability derivatives were for a C-135A aircraft in landing configuration with a steady-state airspeed of 261.8 ft/s and gross weight of 160,000 lb. Results are shown in Fig. 1. Full analysis of the state and control vectors is presented in Ref. 6. The simulation indicated that the optimal control gains based on range provide smooth flare trajectories with a longitudinal touchdown dispersion of only 71.1 ft and a sink rate dispersion at touchdown of only 0.17 ft/s. As mentioned previously, the "Speckled Trout" C-135 simulation yielded range and sink rate dispersions of 2017 ft and 5.55 ft/s. Thus, compared to a conventional operational autopilot, the linear optimal control law permits a very significant reduction in dispersions.

It should be noted that the optimal control technique presented here is good for placing an air vehicle at any point in space, with any terminal conditions, with any in-flight constraints, in any deterministic wind conditions; this can be done regardless of the vehicle's initial conditions. Further investigation should analyze the gust response of the system.

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Nonlinear Method for Parameter Identification Applied to a Trajectory Estimation Problem

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Introduction

A PRIMARY objective of vehicle flight testing is to determine the aerodynamic characteristics of the vehicle. In some cases, the characteristic being sought is a parameter – for example, a stability derivative at a given Mach number. In other cases, it can be a function of one or more variables – for example, the nose tip shape of an ablating re-entry vehicle. The identification of a function can be reduced to the identification of parameters by letting the parameters be values of the function at a number of points and using curve fitting to form the function.

The parameters are identified by minimizing the least-squares performance index, using a variable-metric optimization algorithm and numerical partial derivatives. Hence, only the model defining the motion and initial guesses of the parameters are needed to do the parameter estimation. This procedure has been developed because it eliminates the setup time involved with using the linearized equations of motion characteristic of most estimation algorithms. Because of this feature, this approach can be useful for experimenting with modeling.

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Parameter Estimation Problem

In general, the parameter estimation problem for a dynamical system consists of finding the values of a set of p constants C and n initial states X_0 (assuming $t_0 = 0$) such that the solution of the dynamical system

$$\dot{X} = F(t,X,C) \tag{1}$$

fits the experimental data. The data are fit by minimizing the weighted least-squares performance index

$$J = \sum_{k=1}^{\ell} (Y_k - G_k)^T W (Y_k - G_k) \tag{2}$$

where the subscript k denotes the time at which the experimental data are taken, and W is a positive-definite diagonal matrix whose elements are the inverses of the measurement covariance. The quantity Y is a q vector of the experimental values, and G is a q vector of computed values which is related to the state of the dynamical system, that is, $G = G(t,X)$.

Linear Parameter Estimation

One approach to the solution of this problem is to assume that the change in the parameter vector

$$\beta = [X_0 \ \ C]^T \tag{3}$$

is small. Then, Eqs. (1) and (2) are linearized, and the minimization problem can be solved analytically (see, for example, Ref. 1). Implementation of the method, however, requires the integration of the linear system of differential equations corresponding to Eq. (1) and the inversion of what can be a large-dimension matrix. The linearized approach requires a considerable amount of setup, coding, and debugging time. Also, once a running program has been obtained, it is very time-consuming to change the mathematical model. This makes it difficult to experiment with modeling. The matrix inversion required with this method can also present some problems. This is particularly true if there is insufficient information available to extract a particular parameter.

Nonlinear Parameter Estimation

The method being proposed here eliminates the need for linear system equations and matrix inversion. In terms of the parameter vector β, the problem is to minimize the performance index $J = J(\beta)$, which is obtained by combining Eqs. (1) and (2) functionally. An efficient numerical method for solving this problem is the square-root variable-metric method proposed by Williamson (Ref. 2), which is described by the following algorithm:

1) Guess β and a positive-definite, symmetric matrix S. Compute $J(\beta)$, $J_\beta(\beta)$, and $H = SS^T$.

2) Compute the change in β from the formula

$$\Delta\beta = -\alpha H J_\beta^T \tag{4}$$

using a one-dimensional search to determine the scalar step size α which minimizes J in the $-HJ_\beta^T$ direction. This search yields a new function value $\bar{J}$, where $\bar{J} = J(\beta + \Delta\beta)$.

3) Compute a new gradient $\bar{J}_\beta^T$ and form the difference

$$\Delta J_\beta^T = \bar{J}_\beta^T - J_\beta^T$$

4) Compute a new metric $\bar{H}$ from the relation

$$\bar{H} = \bar{S}\bar{S}^T \tag{5}$$

where

$$\bar{S} = S[I - (\nu/A) S^T Y Y^T S]$$

$$A = Y^T H \Delta J_\beta^T, \quad Y = \alpha J_\beta^T + \Delta J_\beta^T$$

$$\nu = [1 + (1 - B/A)^{1/2}]/(B/A), \quad B = Y^T S S^T Y \tag{6}$$

If ν is imaginary, use $\nu = 1$ in the above expression for $\bar{S}$.

5) If the process has not converged, repeat the procedure. Convergence has occurred when $\alpha \to 1$, $A \to 0$, and $B \to 0$ simultaneously.

Regardless of the accuracy with which the matrix S is computed, the metric H will be positive-definite, and a decrease in the performance index will be achieved on every iteration.

To implement the variable-metric method, one needs to compute $J(\beta)$ and $J_\beta(\beta)$. The performance index is computed as follows:

1) Given $\beta = [X_0 \ \ C]^T$, compute $Y_0 - G(t_0, X_0)$.

2) Integrate $\dot{X} = F(t,X,C)$ from t_0 to t_1; compute $Y_1 - G(t_1, X_1)$. Continue this step until the final time (last observation) is reached.

3) Compute $J(\beta)$ using Eq. (2).

The gradient is obtained by differencing values of the performance index resulting from perturbed values of individual parameters. Here, the central difference formula

$$J_{\beta_j} = [J(\beta_j + \delta\beta_j) - J(\beta_j - \delta\beta_j)]/(2\delta\beta_j) \tag{7}$$

is used where $\delta\beta_j$ is the perturbation in β_j. This formula has a truncation error of $0(\delta\beta_j^2)$, but $\delta\beta_j$ cannot be made too small because of round-off error.

Example

To illustrate the results which can be achieved with this method, the initial conditions and the drag coefficient history of a re-entry vehicle are determined using simulated radar data.

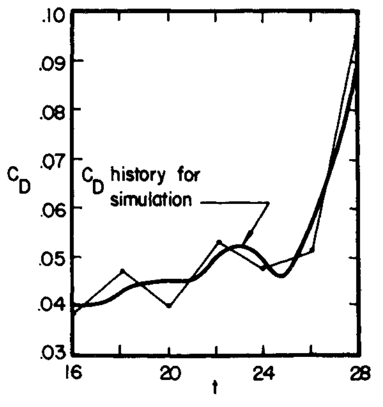

Fig. 1 Linear interpolation of drag coefficient for example problem.

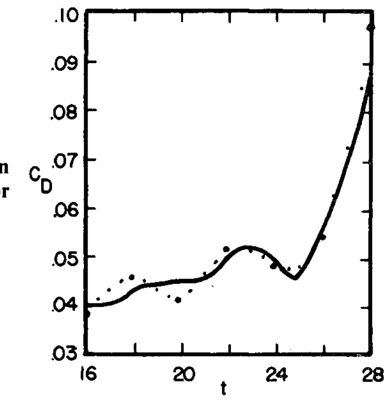

Fig. 2 Spline interpolation of drag coefficient for example problem.

The physical model is composed of the point mass equations of motion relative to a rotating Earth. The Earth is assumed to be an oblate spheroid with a standard atmosphere characteristic of the region where the radar station is located.

To generate the radar data, initial conditions and a drag coefficient history are prescribed. Since the drag coefficient cannot be estimated in regions of extremely low atmospheric density, it has been assumed to be constant over the first part of the trajectory and given by Fig. 1 over the remaining part. The equations of motion are integrated, and radar data in the form of range, azimuth, and elevation are produced at one-second intervals. Finally, six figures of the range, azimuth, and elevation are used as the simulated radar data.

The parameter identification problem consists of estimating the six initial conditions and the value of the drag coefficient at seven points (every 2 s) over the last part of the trajectory. Two methods have been used to interpolate the drag coefficient table: linear interpolation (Fig. 1), and cubic-spline interpolation assuming that the second derivative is zero at the ends (Fig. 2). The initial guess for the vehicle position vector is obtained from the first radar observation, and that for the velocity vector is derived from the first two radar observations. The drag coefficient history is assumed to be constant at the value $C_D = 0.05$.

For both interpolation methods, the initial conditions are extracted to at least five significant figures. On the other hand, the cubic spline interpolation (Fig. 2) allowed a slightly better estimation of the drag coefficient history than the linear fit (Fig. 1). These figures also show that it is difficult to estimate C_D through the first 21 s of the trajectory. This conclusion is supported by the velocity history, which shows very little change through the same time period.

With regard to the performance of the optimization algorithm, convergence is achieved in the manner described earlier. Also, approximately 40 interactions are required to achieve convergence, and the process requires approximately 5 s per iteration.

Conclusions

The nonlinear estimation method proposed here, that is, the weighted least-squares performance index minimized by the square-root variable-metric method using numerical partial derivatives, can be used to obtain reasonable estimates of trajectory characteristics with a minimum of time and effort. Hence, this procedure can be useful for experimenting with modeling. Also, the results of this method can be used as input to more sophisticated estimation algorithms.

Acknowledgment

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